Mathematical Models

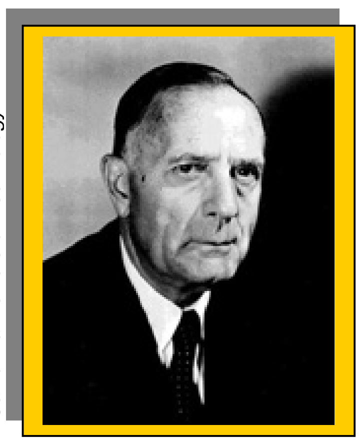
Cosmic Chemistry: Cosmogony

TEACHER GUIDE

BACKGROUND INFORMATION

Science is filled with models. Appendix B, “Assumptions, Models, and the Scientific Method,” includes a section on models and their development. A good example of a model is the Bohr model of the atom, which is introduced to students in elementary chemistry and physics classes. The science of cosmology is no different from the other sciences in its dependence on models. In fact, cosmology may be more model-dependent than are the other sciences because of its relative infancy as a discipline and the difficulty of obtaining large quantities of reliable, relevant data.

The universe as a whole has been modeled through the years, with modern models tracing their origins back to 1917 when Einstein published his theory of general relativity. Assuming the universe is homogeneous and does not change in time, Einstein used his new mathematical theory to develop a complex mathematical model of the overall structure of the universe. Since Einstein’s work was published, there have been numerous new models or modifications of old models. Thus, we have the big bang model, the steady state model, and the inflationary universe model. All of these models are very complex in their details and are beyond the understanding of all except professional cosmologists. Nevertheless, it is important for students to develop an appreciation for mathematical modeling, since this is largely the character of the modeling in cosmology.



California Institute of Technology

American astronomer Edwin Hubble lived between 1889-1953. He was the first person to identify galaxies beyond our own Milky Way. He also observed that these galaxies are moving away from each other, an indication that the universe is expanding.

In the initial activity of this section, students will collect data relating to the way in which points on an elastic material move as the material is stretched. It will be necessary for the students to have a good command of the concept of rate (or speed), that is: distance traveled/time of travel. Once the students understand rate, they will then graph their experimental data and be able to demonstrate that the points on the stretched material behave just as does the expanding universe, as expressed through what is commonly called “Hubble’s Law” (see Appendix A, “Mathematical Models”).

The students then will develop a mathematical statement that describes the physical situation encountered in their experiment. To do this students must know the equation for a straight line, $y = mx + b$. The focus developed in the first activity, that is, analyzing data so that it can be graphed as a straight line, will be reinforced in Parts 2, 3, and 4 of the activity.

Specifically, in Part 2 they will be given solubility data that can be analyzed (with guidance) in terms of an equation involving squared and cubed terms. For Parts 3 and 4 they will need to have a knowledge of logarithms. As the students progress through the four parts, the level of mathematical sophistication required will increase. You will be able to decide how many parts of the activity are suitable for your students. You may elect to have students do only Part 1. This is acceptable, as the idea of mathematical modeling is well illustrated in that exercise.

NATIONAL SCIENCE STANDARDS ADDRESSED

Grades5-8

Science As Inquiry

Understandings about scientific inquiry

Physical Science

Properties and changes of properties in matter

Motion and Forces

History and Nature of Science
Nature of science and scientific knowledge

Grades 9-12
Science As Inquiry
Understandings about scientific inquiry
Physical Science
Motion and forces
Earth and Space Science
The origin and evolution of the universe
History and Nature of Science
Nature of science and scientific knowledge
Historical perspectives

(View a full text of the National Science Education Standards.)

Teaching Tip

The students can mark the initial points on the elastic with ink, but this is not the best way to do it because the ink marks expand during the stretching of the material. Thus, it becomes necessary to estimate the position of the center of the point on the stretched material and this introduces errors. A much better way to mark the points is with a staple attached to the edge of the elastic or with thumb tacks pushed through the elastic. If thumb tacks are used, students should stretch the material with the tack heads underneath the elastic, so that the position of the stem of the tacks can be recorded. Make sure that the students mark and measure the location of the initial points on unstretched elastic that has been laid out flat and straight. If the elastic has been coiled up, it may be a good idea for you to uncoil and hang the elastic for a few days with a small weight on the end in order to straighten it out.

The students should collect data for a minimum of 10 points on the elastic. The points should be distributed randomly over about 40-50 cm of the initial 60 cm length of elastic. This is a much more convincing experiment if some points are close and some far apart initially.

MATERIALS

For each student:
- Copy of Student Activity, "Mathematical Models"
- Copy of Appendix A, "Cosmology"
- Copy of Appendix B, "Assumptions, Models, and The Scientific Method"

For each team: (Measurements are approximate.)
- Several sheets of graph paper.
- A 60-cm length of ¼ or ½-inch wide elastic sewing material.
- A flat surface along which the elastic can be stretched and held in place while measurements are made. A small wooden board 1.5 inches wide, 0.75 inches thick and 36 inches long is convenient, but not necessary. (36 inches of a 1 x 2)
- Two clamps for clamping the ends of the elastic in place after it has been stretched. Small "C" clamps or binder clips can be used, depending on the surface along which the elastic is stretched. Thumbtacks might also be used.
- Two meter sticks. During pilot testing, students found that placing two meter sticks end-to-end with the stretch material placed on top eased measuring. Rulers will work, but their use introduces errors because of the necessity of repeatedly moving the ruler to measure distances of 100 cm or more. If rulers are used, emphasize the need to make very careful measurements.

Other items for the entire class:
- Thumb tacks or straight pins.

PROCEDURE

1. Before class make copies of the following:
   Student Activity, "Mathematical Models"
   Appendix A, "Cosmology"
   Appendix B, "Assumptions, Models, and the Scientific Method"

2. Distribute copies of the Student Activity and Appendices.

3. Divide the class up into teams consisting of 3 students and ask them to read Appendix B, "Assumptions, Models, and the Scientific Method" for Part 1. For Part 2 you may again form student teams or have the students work on an individual basis.

4. Distribute several sheets of graph paper to each team or student.

PART 1

5. The students will devise an experiment in which they measure the rate at which points on an elastic material move as the material is stretched. The rate of movement is not obvious to most people.

6. Discuss in detail the concept of rate, emphasizing that it is a **derived quantity** stemming from measurements of distance and time. A graph showing rate on the y-axis will be plotted by the students. Rate is the measured distance (D) that a given point moves divided by the time (T) of the movement. It is necessary for students to grasp the concept, and this is a bit subtle, that the TIME required for the movement of every point from its initial location to its final (stretched) location IS THE SAME. Since the time is the same for all points, their relative rates are simply proportional to the distance traveled. They will graph on the y-axis the quantity D/T, with time undetermined, but constant, for all points. Therefore, the units on the y-axis will just be multiples of 1/T. For example, they might find that two points move 10 and 15 cm, respectively. On their graph they will plot the rates as 10/T and 15/T for each of the points. It is critically important for them to understand that this is a relative rate.

7. Each team should submit a written description, complete with diagrams, of the experimental setup that they will employ. Make sure they submit a diagram showing where they think points would be located after the elastic is stretched.

   Post the descriptions and predictions around the room for all students to read.

8. After the students have devised and submitted an experimental procedure, but before they actually conduct the experiment, bring the class together for a discussion of the various procedures that have been proposed. It would be appropriate to discuss the following questions:
   a) Would the “stretchiness” of the elastic have any effect on the results?
   b) Would it be possible to use a rubber band instead of elastic? What would be the disadvantages of using a rubber band?
   c) Would the width of the elastic have any effect on the results?
   d) Is the slope of the line on the graph dependent on whether one plots absolute rate or relative rate?

9. Give each team a chance to modify their proposed procedure, if necessary, and then tell them to conduct the experiment that they have designed. Ask them to prepare a report on their findings and submit it to you. As part of the report, they should submit their graph, with axes clearly labeled. If students are having trouble constructing the y-axis, refer to the top teaching tip. They should also comment in the report on how their prediction about the movement of the points compares to their experimental findings. Post the graphs around the room so that the results can be compared.

10. After the students have completed their work, assign Appendix A, “Cosmology” to be read. Engage the students in a discussion organized around questions like the following:

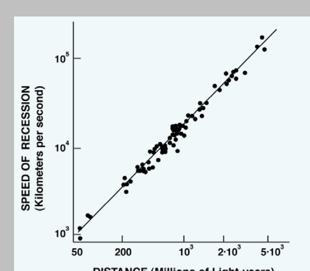
Teaching Tip

Students have difficulty with the concept of relative rate. They may be more comfortable actually measuring the time required to stretch the elastic. In this case, simply have them stretch the elastic slowly and use a stop-watch, to time the stretching motion. Then they can actually calculate a rate for the movement of the points in, perhaps, cm/s. This would then be an “absolute” rate, and not a relative rate.

You could then use this as an opportunity to establish the fact that the effect of including time is only to move the points up or down on the y-axis. In other words, if one compares a point that moves 20 cm during one second (rate = 20/1 = 20 cm/s) to one that moves 20 cm in 10 seconds (rate=20/10=2 cm/s), one sees immediately that the first point’s location on the y-axis is just 10 units higher than the second point’s location. The effect of including a constant time factor is simply to move points uniformly up and down the y-axis.

Teaching Tip

If students do not know how to construct a best fit graph, provide an example such as the one shown here:



Teaching Tip

During pilot testing, teachers found that student groups of three worked best.

- 1 to hold reference point.
- 1 to stretch.
- 1 to measure and record.

a) Could they use ANY point as a reference, or does the reference point have to be on the end of the elastic?
b) Is the rate of movement the same at all points along the elastic? Is it the case that the rate of any point is proportional to its distance from the reference point? In other words, is it true that a point initially at 4 cm moves twice as fast as a point initially at 2 cm?
c) How is m, the slope of the line, related to your answers in question b?
d) What would be the slope of the line if all points moved at the same rate, independent of their initial position? If points moved at a rate proportional to three times their initial distance from the reference point?
e) How do their results compare to the results presented in the section on Hubble's Law in Appendix A, "Cosmology," for the universe?
f) Is stretched material a good model for the expansion of the universe?

Teaching Tip

A more realistic model of the expansion of the universe can be demonstrated with a balloon. You may want to ask the students to devise an experiment in which they mark points on a partially inflated balloon and then make measurements on how the distances between the points change as the balloon is inflated. The points on the balloon represent clusters of galaxies. It is particularly instructive for students to make measurements using several different points as reference points. They will see that the results are independent of their choice. There is no unique point, just as is the case with the universe.

PART 2

11. This activity may be stopped with the completion of Part 1, depending on the mathematical sophistication of your students. However, should you wish to have students advance the idea of mathematical modeling, which, of course, is so important to cosmology, then they can productively pursue the three exercises in Part 2. If the students are not familiar with logarithms, you will want to have them complete only section a).

12. If you have the students pursue Part 2, begin with a class review of why it is so useful and convenient to derive models based on straight-line graphs. Emphasize the predictive power of mathematical models. Also, emphasize the need to construct graphs that have scales on the abscissa and ordinate that utilize most of the graph paper. They should not submit graphs in which the data are squished up in one corner. Review the concept of the slope of a line and how to determine slope. Make sure they understand that slopes may be positive or negative.

13. In order to complete sections b) and c) of Part 2, it is necessary for students to be familiar with logarithms. Review this concept if necessary. Make sure they understand the concept that an equation of the form log Q = mlog W + B, would provide a straight line if one plots log Q versus log W.

14. After the students have completed one or more of these sections, post their graphs and equations.

Teaching Tip

Equations for Student Activity Part 2:

For section a) the equation is:
$S = 0.5T + 0.002T^2$.

For section b) the equation is:
$\log P = 3.50 - 0.110A$

For section c) the equation is:
$\log T = -0.7 + 1.5\log D$.

From this equation, taking antilogs, one obtains $T = 0.2D^{1.5}$, or $T^2 \propto D^3$.

15. Call the students together as a group and have a discussion that focuses on questions such as:
a) What things might they consider modeling mathematically in their everyday life? Maybe gallons of gasoline used versus miles driven in an automobile? Maybe the reproductive rate of bacteria? Maybe the rate at which a student's heart beat returns to normal after vigorous exercise? Some of the ideas they have may be ones that they can actually investigate. Be careful though. Not everything can be modeled as a straight-line function. Some of their ideas may lead to complex mathematics!
b) What type of graph would one obtain if one were to plot P versus log A in Part 2b of the student activity?
c) What would be the sidereal period of planet X, if we assume that this planet is located 6.5×10^9 km from the sun?